Please check the examination details below before entering your candidate information

Candidate surname	Other names

Centre Number

Candidate Number

Pearson Edexcel Level 3 GCE

Wednesday 14 June 2023

Afternoon (Time: 1 hour 30 minutes)

Paper reference **9FM0/3C**

Further Mathematics

Advanced

PAPER 3C: Further Mechanics 1

You must have:
Mathematical Formulae and Statistical Tables (Green), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Unless otherwise indicated, whenever a value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 7 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over

Pearson

1. A particle P of mass 2 kg is moving with velocity $(-4\mathbf{i} + 3\mathbf{j})$ m s^{-1} when it receives an impulse $(-6\mathbf{i} + 42\mathbf{j})$ N s.

(a) Find the speed of P immediately after receiving the impulse.

(4)

The angle through which the direction of motion of P has been deflected by the impulse is $\alpha°$

(b) Find the value of α

(2)

a) Using $\underline{I} = m(\underline{v} - \underline{u})$

$$\begin{pmatrix}-6\\42\end{pmatrix} = 2\left(\underline{v} - \begin{pmatrix}-4\\3\end{pmatrix}\right) \quad 1 \quad 1$$

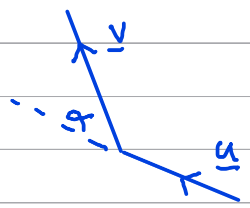
$$\underline{v} = 0.5\begin{pmatrix}-6\\42\end{pmatrix} + \begin{pmatrix}-4\\3\end{pmatrix}$$

$$\underline{v} = \begin{pmatrix}-7\\24\end{pmatrix}$$

1

$$\text{speed} = \sqrt{(-7)^2 + 24^2} = 25\,\text{ms}^{-1} \quad 1$$

b)



using dot product: $\cos\alpha = \dfrac{\begin{pmatrix}-4\\3\end{pmatrix} \cdot \begin{pmatrix}-7\\24\end{pmatrix}}{5 \times 25}$ 1

$\sqrt{(-4)^2 + 3^2} = 5$

$$\cos\alpha = 0.8$$

$$\alpha = 37° \text{ (2sf)} \quad 1$$

Question 1 continued

(Total for Question 1 is 6 marks)

2. A car of mass 1000 kg moves in a straight line along a horizontal road at a constant speed U m s^{-1}. The resistance to the motion of the car is a constant force of magnitude 400 N.

The engine of the car is working at a constant rate of 16 kW.

(a) Find the value of U.

(3)

The car now pulls a trailer of mass 600 kg in a straight line along the road using a tow rope which is parallel to the direction of motion. The resistance to the motion of the car is again a constant force of magnitude 400 N. The resistance to the motion of the trailer is a constant force of magnitude 300 N.

The engine of the car is working at a constant rate of 16 kW.

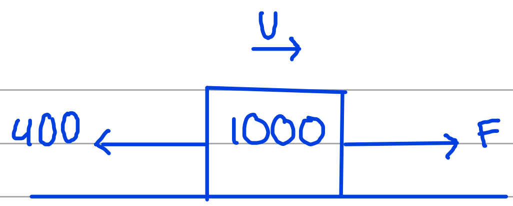
The tow rope is modelled as being light and inextensible.

Using the model,

(b) find the tension in the tow rope at the instant when the speed of the car is $\frac{20}{3}$ m s^{-1}

(5)

a)



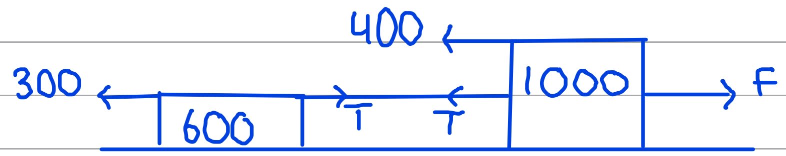
$P = 16{,}000 = FU \Rightarrow F = \dfrac{16{,}000}{U}$ (1)

Resolve $(\rightarrow +)$: $F - 400 = 0$ (1)

$F = 400$

$400 = \dfrac{16{,}000}{U} \Rightarrow U = 40$ (1)

b)



$P = 16{,}000 = \dfrac{20}{3} F \Rightarrow F = 2400$ (1)

(1)

Resolve $\rightarrow +$ for car: $F - T - 400 = 1000a$ (1)

$2000 - T = 1000a$ (1)

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Question 2 continued

Resolve $\rightarrow +$ for trailer: $T - 300 = 600a$ (2) (1)

(1) + (2): $2000 - 300 = 1000a + 600a$

$a = 1.0625 \text{ ms}^{-2}$

sub into (2): $T = 300 + 600(1.0625)$

$= 937.5 \text{ N}$ (1)

Question 2 continued

Question 2 continued

(Total for Question 2 is 8 marks)

3. A particle P of mass $2m$ is moving in a straight line with speed $3u$ on a smooth horizontal plane. It collides directly with a particle Q of mass m that is moving on the plane with speed $2u$ in the opposite direction to P.

The coefficient of restitution between P and Q is e, where $e > \frac{4}{5}$

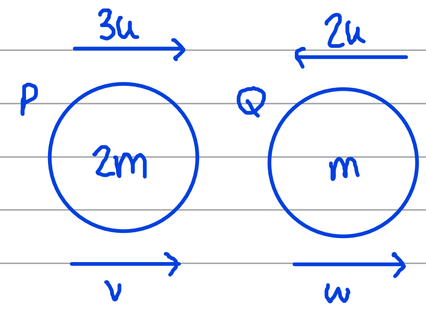
(a) Show that the speed of Q immediately after the collision is $\frac{(4 + 10e)u}{3}$ **(6)**

After the collision Q hits a smooth fixed vertical wall that is perpendicular to the direction of motion of Q. The coefficient of restitution between Q and the wall is f.

(b) Find, **in terms of e**, the set of values of f for which there will be a second collision between P and Q.

(4)

a)



CLM $(\rightarrow +)$: $3u(2m) - 2um = 2mv + mw$ (1) (1)

$2v + w = 4u$ (1)

NLR: $e = \dfrac{w - v}{3u + 2u}$ (1) $\Rightarrow$ $w - v = 5eu$ (1)

$2w - 2v = 10eu$ (2)

(1) + (2): $2v + w + 2w - 2v = 4u + 10eu$ (1)

$3w = (4 + 10e)u$

$w = \dfrac{(4 + 10e)u}{3}$ as required. (1)

b) Method: find v and the speed of Q after it hits the wall (w') and set w' > v

from (2): $v = w - 5eu$

$= \dfrac{4u}{3} + \dfrac{10eu}{3} - 5eu$

$= \dfrac{(4 - 5e)u}{3}$ (1)

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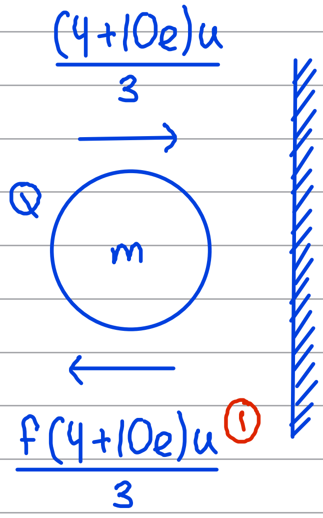
Question 3 continued

since $e > \frac{4}{5}$, $4 - 5e < 0$, so $v < 0$. we are considering the positive speed of P, which is $-\frac{(4-5e)u}{3}$



for second collision, $\frac{f(4+10e)u}{3} > -\frac{(4-5e)u}{3}$ 1

$$f(4+10e) > 5e - 4$$

$$f > \frac{5e-4}{4+10e}$$

$$\therefore \frac{5e-4}{4+10e} < f \le 1$$ 1

Question 3 continued

Question 3 continued

(Total for Question 3 is 10 marks)

4. A light elastic string has natural length $2a$ and modulus of elasticity $4mg$.
One end of the elastic string is attached to a fixed point O. A particle P of mass m is attached to the other end of the elastic string.
The particle P hangs freely in equilibrium at the point E, which is vertically below O

(a) Find the length OE.

(4)

Particle P is now pulled vertically downwards to the point A, where $OA = 4a$, and released from rest. The resistance to the motion of P is a constant force of magnitude $\frac{1}{4}mg$.

(b) Find, in terms of a and g, the speed of P after it has moved a distance a.

(7)

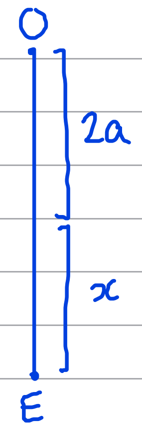
Particle P is now held at O
Particle P is released from rest and reaches its maximum speed at the point B.

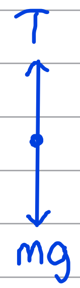
The resistance to the motion of P is again a constant force of magnitude $\frac{1}{4}mg$.

(c) Find the distance OB.

(4)

a) string
$l = 2a$
$\lambda = 4mg$





Resolve ($\uparrow +$): $T = mg$ ①

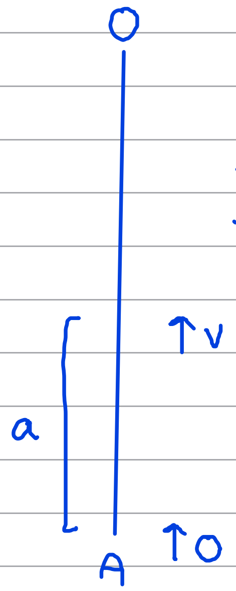
$T = \frac{\lambda x}{l} = \frac{4mgx}{2a}$ ① $\frac{4mgx}{2a} = mg$

$x = \frac{2amg}{4mg} = \frac{1}{2}a$ ①

$OE = 2a + \frac{1}{2}a = \frac{5}{2}a$ ①

Question 4 continued

b)



$$\begin{aligned}&\text{KE before} &&\text{KE after}\\ +\ &\text{GPE before} \quad = &&+\ \text{GPE after}\\ +\ &\text{EPE before} &&+\ \text{EPE after}\\ & &&+\ \text{work done against resistance}\end{aligned}$$

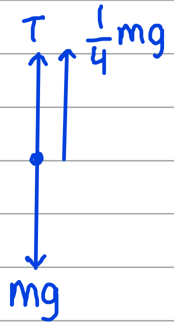
$$0 + 0 + \frac{4mg(2a)^2}{2 \times 2a} \ (1) = \frac{1}{2}mv^2 + mga \ (1) + \frac{4mga^2}{2 \times 2a} \ (2) + \frac{1}{4}mga \ (1)$$

$$4mga = \frac{1}{2}mv^2 + mga + mga + \frac{1}{4}mga \ (1)$$

$$\frac{1}{2}v^2 = 4ga - ga - ga - \frac{1}{4}ga$$

$$v = \sqrt{\frac{7ag}{2}} \ (1)$$

c) highest speed when $a = 0$



$a = 0$: Resolve (↑) $\quad T + \frac{1}{4}mg = mg$ (1)

$$T = \frac{3mg}{4}$$

$$\frac{3mg}{4} = \frac{4mgx}{2a} \ (1)$$

Question 4 continued

$2x = \frac{3a}{4} \Rightarrow x = \frac{3a}{8}$ (1)

$OB = 2a + \frac{3a}{8} = \frac{19a}{8}$ (1)

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Question 4 continued

(Total for Question 4 is 15 marks)

5.

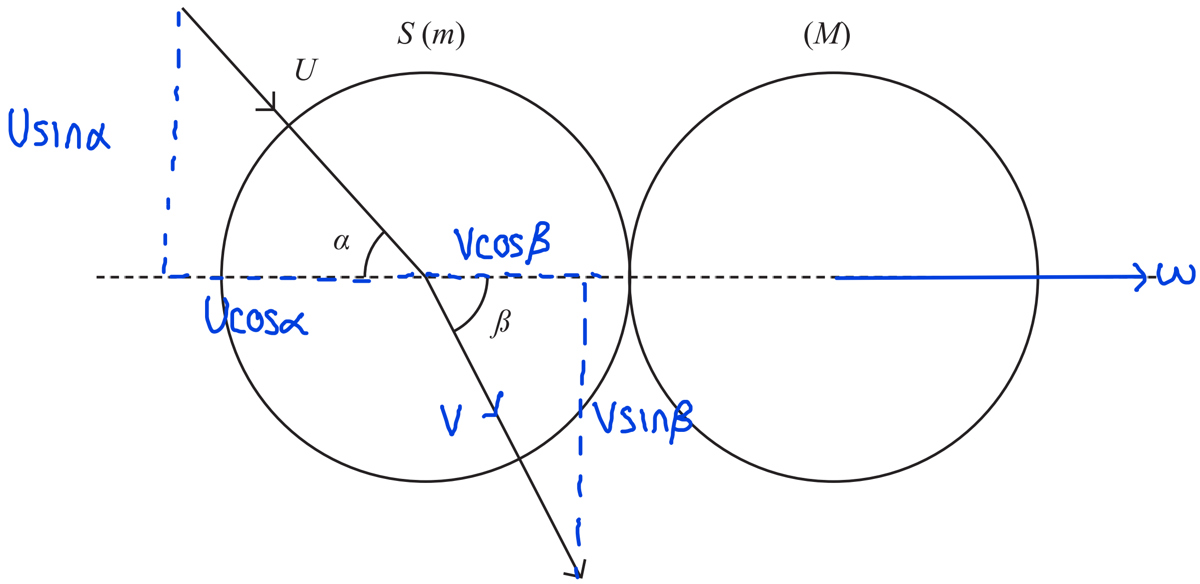

Figure 1

A smooth uniform sphere S of mass m is moving with speed U on a smooth horizontal plane. The sphere S collides obliquely with another uniform sphere of mass M which is at rest on the plane. The two spheres have the same radius.

Immediately before the collision the direction of motion of S makes an angle α, where $0 < \alpha < 90°$, with the line joining the centres of the spheres.

Immediately after the collision the direction of motion of S makes an angle β with the line joining the centres of the spheres, as shown in Figure 1.

The coefficient of restitution between the spheres is e.

(a) Show that $\tan\beta = \dfrac{(m + M)\tan\alpha}{(m - eM)}$ **(8)**

Given that $m = eM$,

(b) show that the directions of motion of the two spheres immediately after the collision are perpendicular. **(2)**

a) $\perp$ to LOC:

$U\sin\alpha = V\sin\beta$ $\boxed{A}$ 1

// to LOC:

CLM($\rightarrow +$): $mU\cos\alpha + 0 = mV\cos\beta + Mw$ 1 1 1

NLR: $e = \dfrac{w - V\cos\beta}{U\cos\alpha}$ 1 $\Rightarrow eU\cos\alpha = w - V\cos\beta$ 1

$MeU\cos\alpha = Mw - MV\cos\beta$ 2

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Question 5 continued

$① - ②:\ mU\cos\alpha - MeU\cos\alpha = mV\cos\beta + MV\cos\beta$

$U\cos\alpha(m - eM) = V\cos\beta(m + M)\ \boxed{B}$

$\boxed{A} \div \boxed{B}:\ \dfrac{U\sin\alpha}{U\cos\alpha(m-eM)} = \dfrac{V\sin\beta}{V\cos\beta(m+M)}$ (1) (1)

$\dfrac{\tan\alpha}{m-eM} = \dfrac{\tan\beta}{m+M}$

$\tan\beta = \dfrac{(m+M)\tan\alpha}{m-eM}$ as required (1)

b) $m = eM$ so $m - eM = 0$, $\tan\beta$ is undefined $\Rightarrow \beta = 90°$ (1)

Hence, after the collision S moves perpendicular to the line of centres (LOC) and the other sphere moves parallel to the LOC so they move at right angles to each other. (1)

Question 5 continued

Question 5 continued

(Total for Question 5 is 10 marks)

6. A particle P of mass m is falling vertically when it strikes a fixed smooth inclined plane. The plane is inclined to the horizontal at an angle α, where $0 < \alpha \leqslant 45°$

At the instant immediately before the impact, the speed of P is u.

At the instant immediately after the impact, P is moving horizontally with speed v.

(a) Show that the magnitude of the impulse exerted on the plane by P is $mu\sec\alpha$ **(5)**

The coefficient of restitution between P and the plane is e, where $e > 0$

(b) Show that $v^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha)$ **(3)**

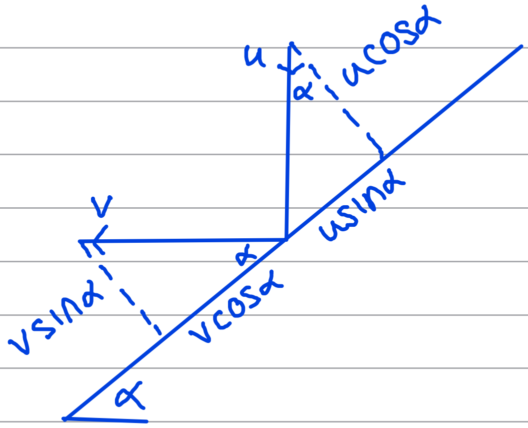
(c) Show that the kinetic energy lost by P in the impact is

$$\frac{1}{2}mu^2(1-e^2)\cos^2\alpha$$

(2)

(d) Hence find, in terms of m, u and e **only**, the kinetic energy lost by P in the impact. **(2)**

a)



CLM along the plane (↗ +): $vm\cos\alpha = um\sin\alpha$ ①①

$V = \dfrac{u\sin\alpha}{\cos\alpha}$

considering impulse (↖ +): $I = m(v\sin\alpha + u\cos\alpha)$ ①①

$= m\left(\dfrac{u\sin^2\alpha}{\cos\alpha} + u\cos\alpha\right)$

$= \dfrac{mu}{\cos\alpha}(\sin^2\alpha + \cos^2\alpha)$

$= mu\sec\alpha$ ①

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Question 6 continued

b) NLR: $e = \dfrac{v\sin\alpha}{u\cos\alpha} \Rightarrow eu\cos\alpha = v\sin\alpha$ 1

square both sides: $v^2\sin^2\alpha = e^2u^2\cos^2\alpha$ ①

also from CLM: $v\cos\alpha = u\sin\alpha$

square both sides: $v^2\cos^2\alpha = u^2\sin^2\alpha$ ②

① + ② : $v^2\sin^2\alpha + v^2\cos^2\alpha = e^2u^2\cos^2\alpha + u^2\sin^2\alpha$ 1

$$v^2(\sin^2\alpha + \cos^2\alpha) = u^2(e^2\cos^2\alpha + \sin^2\alpha)$$

$$v^2 = u^2(\sin^2\alpha + e^2\cos^2\alpha)$$ 1

c) KE loss = KE before - KE after

$$= \frac{1}{2}mu^2 - \frac{1}{2}mv^2$$

$$= \frac{1}{2}mu^2 - \frac{1}{2}m\left[u^2(\sin^2\alpha + e^2\cos^2\alpha)\right]$$ 1

$$= \frac{1}{2}mu^2 - \frac{1}{2}mu^2(1 - \cos^2\alpha + e^2\cos^2\alpha)$$

$$= \frac{1}{2}mu^2 - \frac{1}{2}mu^2 + \frac{1}{2}mu^2\cos^2\alpha - \frac{1}{2}mu^2e^2\cos^2\alpha$$

$$= \frac{1}{2}mu^2(1 - e^2)\cos^2\alpha$$ 1

d) $eu\cos\alpha = v\sin\alpha$ and $u\sin\alpha = v\cos\alpha$

$\therefore \dfrac{eu\cos\alpha}{u\sin\alpha} = \dfrac{v\sin\alpha}{v\cos\alpha} \Rightarrow e = \tan^2\alpha$

using $1 + \tan^2\alpha = \sec^2\alpha$:

Question 6 continued

$\sec^2\alpha = 1 + e$

$\cos^2\alpha = \dfrac{1}{1+e}$ 1

so KE lost $= \dfrac{1}{2} mu^2 (1+e)(1-e) \times \dfrac{1}{1+e}$

$= \dfrac{1}{2} mu^2 (1-e)$ 1

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Question 6 continued

(Total for Question 6 is 12 marks)

7.

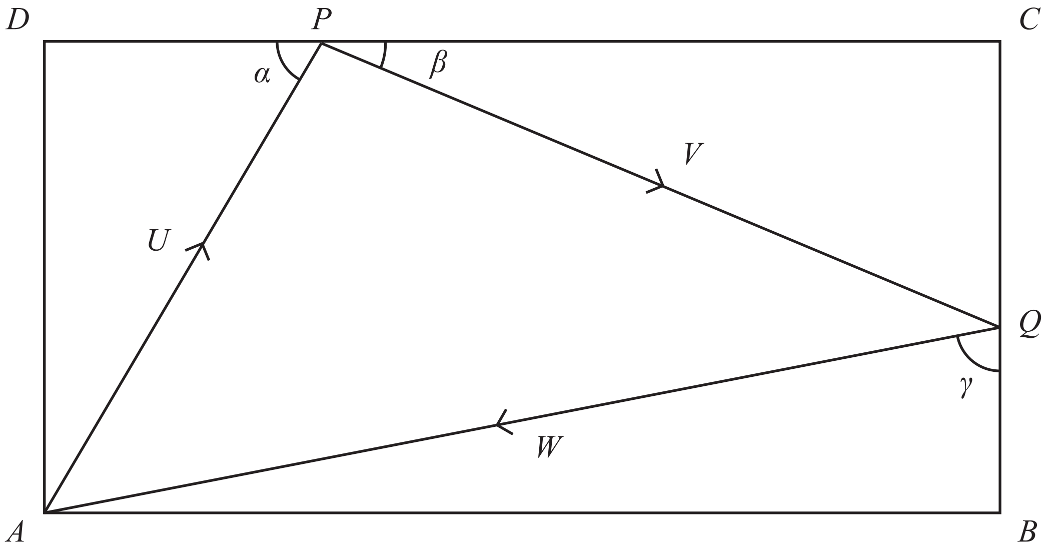

Figure 2

A small smooth snooker ball is projected from the corner A of a horizontal rectangular snooker table $ABCD$.

The ball is projected so it first hits the side DC at the point P, then hits the side CB at the point Q and then returns to A.

Angle $APD = \alpha$, Angle $QPC = \beta$, Angle $AQB = \gamma$
The ball moves along AP with speed U, along PQ with speed V and along QA with speed W, as shown in Figure 2.

The coefficient of restitution between the ball and side DC is e_1

The coefficient of restitution between the ball and side CB is e_2

The ball is modelled as a particle.

Use the model to answer all parts of this question.

(a) Show that $\tan\beta = e_1 \tan\alpha$

(4)

(b) Hence show that $e_1 \tan\alpha = e_2 \cot\gamma$

(3)

(c) By considering (angle APQ + angle AQP) or otherwise, show that it would be possible for the ball to return to A only if $e_2 > e_1$

(6)

If instead $e_1 = e_2$, the ball would **not** return to A.

Given that $e_1 = e_2$

(d) use the result from part (b) to describe the path of the ball after it hits CB at Q, explaining your answer.

(1)

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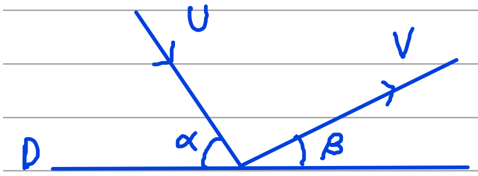
Question 7 continued

a) considering side DC:

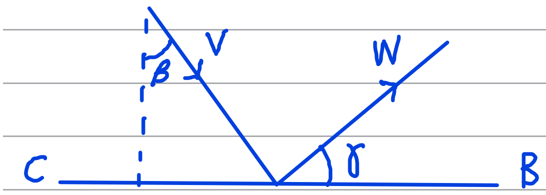


parallel to DC: $U\cos\alpha = V\cos\beta$ ① ①
perpendicular: $e_1 U\sin\alpha = V\sin\beta$ ② ①

② ÷ ①: $\dfrac{e_1 U\sin\alpha}{U\cos\alpha} = \dfrac{V\sin\beta}{V\cos\beta}$ ①

$e_1\tan\alpha = \tan\beta$ as required ①

b) considering side CB:



parallel to CB: $V\sin\beta = W\cos\gamma$ ①
perpendicular: $e_2 V\cos\beta = W\sin\gamma$ ②

② ÷ ①: $\dfrac{e_2 V\cos\beta}{V\sin\beta} = \dfrac{W\sin\gamma}{W\cos\gamma}$

$e_2\cot\beta = \tan\gamma$ ①

$e_2\cot\gamma = \tan\beta$

$e_2\cot\gamma = e_1\tan\alpha$ as required ①

Question 7 continued

c) $\angle APQ = 180 - \alpha - \beta$ and $\angle AQP = 180 - (90 - \beta) - \gamma$

$\angle APQ + \angle AQP = 180 - \alpha - \beta + 180 - 90 + \beta - \gamma$
$= 270 - \alpha - \gamma$ (1)

to return to A, $\angle APQ + \angle AQP < 180$,
since APQ is a triangle (1)

$\therefore 270 - \alpha - \gamma < 180$
$\alpha + \gamma > 90$
$\alpha > 90 - \gamma$ (1)

since α and $90 - \gamma$ are both acute $(0 < \alpha, 90 - \gamma < 90)$
if $\alpha > 90 - \gamma$, $\tan\alpha > \tan(90 - \gamma)$

(1) $\tan\alpha > \tan(90 - \gamma)$ $\tan(90 - x) = \cot x$
$\tan\alpha > \cot\gamma$

$\frac{e_2 \cot\gamma}{e_1} > \cot\gamma$ (1)

$e_2 > e_1$ as required (1)

d) if $e_2 = e_1$, $\alpha = 90 - \gamma$ so the ball would move parallel to the initial velocity (1)

Question 7 continued

Question 7 continued

(Total for Question 7 is 14 marks)

TOTAL FOR PAPER IS 75 MARKS